

Class Problem 10.1 (Collusion) Solution

- Two ready-to-eat breakfast cereal manufacturers, Lots of Sugar and Buckets of Goo, face combined demand for their products given by $Q = 75 - P$. Their total costs are given by $TC_{\text{Lots of Sugar}} = 0.1Q_{\text{Lots of Sugar}}^2$ and $TC_{\text{Buckets of Goo}} = 5Q_{\text{Buckets of Goo}}$. If they successfully collude, their total profits will be _____.

Note that $MC_{\text{Lots of Sugar}} = 0.2Q$ and $MC_{\text{Buckets of Goo}} = 5$; note also that that MC is smaller than 5 for $Q < 10$, but equal to 5 for $Q \geq 10$. Then $MR = 75 - 2Q = 5 = MC \Rightarrow Q = 35$ and $P = 75 - 35 = 40$. Thus Lots of Sugar produces 25 units, Buckets of Goo produces 10 units, and total profit is $40(35) - .1(25^2) - 5(10) = \$1,287.50$.

Class Problem 10.2 (Stackelberg)

Glyde Air Fresheners is the dominant firm in the solid room aromatizer industry which has a total market demand given by $Q = 80 - 2P$. Glyde has competition from a fringe of four small firms that produce where their individual marginal costs equal the market price. The fringe firms each have total costs given by $TC_i = 10Q_i + 2Q_i^2$. If Glyde's total costs are given by $TC_G = 100 + 6Q_G$, what price should Glyde establish for air fresheners?

Class Problem 10.2 Solution

- Here, Glyde is the leader and the followers are the fringe firms. The starting point is to figure out the *reaction function* for the followers, since the leader maximizes profit based upon what she believes is the followers' "best response" to her pricing and output decisions.
- The marginal cost for each of the followers is $MC_i = 10 + 4Q_i$. Let $Q_F = 4Q_i$; i.e., Q_F represents the total production from all four followers. Since $Q = 80 - 2P$, this implies that $P = 40 - 0.5Q = 40 - 0.5Q_F - 0.5Q_G$. Since $P = MC_i$ for the followers, we use this equation to obtain the followers' reaction function; i.e., $P = MC_i \Rightarrow 40 - 0.5Q_F - 0.5Q_G = 10 + Q_F \Rightarrow Q_F = 20 - Q_G/3$.

Class Problem 10.2 Solution

- Next, in order to maximize Glyde's profit, we set marginal revenue equal to marginal cost for Glyde. In order to find Glyde's marginal revenue (MR_G), we calculate total revenue (TR_G), where $TR_G = PQ_G$. Substituting the followers' reaction function into the price equation, we obtain

$$\begin{aligned}P &= 40 - 0.5Q_F - 0.5Q_G \\&= 40 - 0.5(20 - Q_G/3) - 0.5Q_G \\&= 30 - Q_G/3.\end{aligned}$$

- Thus, $TR_G = PQ_G = (30 - Q_G/3)Q_G = 30Q_G - Q_G^2/3 \Rightarrow MR_G = 30 - (2/3)Q_G$. Setting $MR_G = MC_G$, we solve for Q_G :

$$MR_G = MC_G \Rightarrow 30 - (2/3)Q_G = 6 \Rightarrow Q_G = 36.$$

- Since Glyde produces 36 air fresheners, the fringe firms will produce $Q_F = 20 - Q_G/3 = 20 - 12 = 8$ air fresheners. Therefore, total industry output is $Q = Q_F + Q_G = 8 + 36 = 44$, and $P = 40 - 0.5Q = 40 - 22 = \18 .